1. (a) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

(2)

(b) Hence find, for -180° ≤ θ < 180°, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1$$

Give your answers to 1 decimal place.

(3)
(Total 5 marks)

2. (a) Express 2 sin θ – 1.5 cos θ in the form R sin (θ – α), where R > 0 and 0 < α < $\frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of 2 sin θ – 1.5 cos θ.

(ii) Find the value of θ, for 0 ≤ $\theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \le t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)
(Total 15 marks)

3. (a) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\tan^2 \theta = \sec^2 \theta - 1$.

(2)

(b) Solve, for $0 \leq \theta < 360°$, the equation

$$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2$$

(6)
(Total 8 marks)

4. (a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1 \colon y = 3\sin 2x$$

$$C_2 \colon y = 4 \sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R \cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)
(Total 12 marks)

5. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8 \cos x = 0$$

giving your answers to 2 decimal places.

(5)
(Total 6 marks)

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3\theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)
(Total 13 marks)

7. (a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

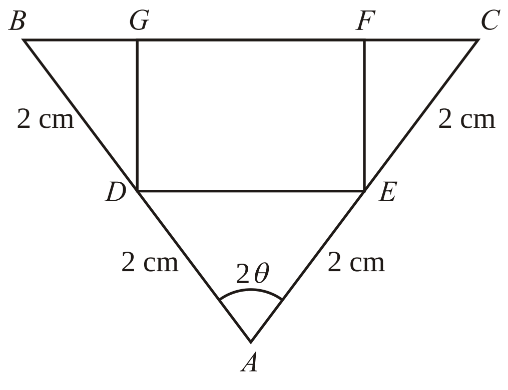
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)
(Total 7 marks)

8.



This diagram shows an isosceles triangle ABC with $AB = AC = 4$ cm and $\angle BAC = 2\theta$.

The mid-points of AB and AC are D and E respectively. Rectangle $DEFG$ is drawn, with F and G on BC. The perimeter of rectangle $DEFG$ is P cm.

(a) Show that $DE = 4\sin\theta$.

(2)

(b) Show that $P = 8 \sin\theta + 4 \cos\theta$.

(2)

(c) Express P in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

Given that $P = 8.5$,

(d) find, to 3 significant figures, the possible values of θ.

(5)
(Total 13 marks)

9. (i) Given that $\sin x = \frac{3}{5}$, use an appropriate double angle formula to find the exact value of $\sec 2x$.

(4)

(ii) Prove that

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \qquad \left(x \neq \frac{n\pi}{2},\ n \in \mathbb{Z}\right).$$

(4)
(Total 8 marks)

10. Find, giving your answers to two decimal places, the values of w, x, y and z for which

(a) $e^{-w} = 4$,

(2)

(b) $\arctan x = 1$,

(2)

(c) $\ln(y + 1) - \ln y = 0.85$

(4)

(d) $\cos z + \sin z = \frac{1}{3}, -\pi < z < \pi.$

(5)
(Total 13 marks)

1. (a) $\dfrac{2\sin\theta\cos\theta}{1 + 2\cos^2\theta - 1}$ M1

$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG** A1 **cso** 2

Note

M1: Uses **both** a correct identity for sin 2θ **and** a correct identityfor cos2θ .
Also allow a candidate writing 1+cos2θ =2cos^2 θ on the denominator.
Also note that angles **must be consistent** in when candidates apply these identities.
A1: Correct proof. No errors seen.

(b) $2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$ M1

$\theta_1 = \text{awrt } 26.6^\circ$ A1

$\theta_2 = \text{awrt } -153.4^\circ$ A1 3

Note

1st M1 for either 2tanθ = 1 or tanθ = $\dfrac{1}{2}$, seen or implied.

A1: awrt 26.6
A1ft: awrt – 153.4° or θ_2 = – 180° + θ_1

Special Case: For candidate solving, tanθ = k, where, $k \neq \frac{1}{2}$, to give θ_1 and θ_2 = – 180° + θ_1, then award M0A0B1 in part (b).
Special Case: Note that those candidates who writes tanθ = 1, and gives ONLY two answers of 45° and –135° that are inside the range will be awarded SC M0A0B1.

[5]

2 (a) $R = \sqrt{6.25}$ or 2.5 B1

$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$ M1 A1 3

Note

B1: R = 2.5 or R = $\sqrt{6.25}$. For R = ± 2.5, award B0.
M1: tan α = ± $\frac{1.5}{2}$ or tan α = ± $\frac{2}{1.5}$
A1: α = awrt 0.6435

(b) (i) Max Value = 2.5 B1ft

(ii) $\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}$;
$\Rightarrow \theta = \text{awrt } 2.21$ M1; A1ft 3

Note

B1$\sqrt{\ }$: 2.5 or follow through the value of R in part (a).
M1: For sin (θ – their α) = 1
A1$\sqrt{\ }$: awrt 2.21 or $\frac{\pi}{2}$ + their α rounding correctly to 3 sf.

(c) $H_{\text{Max}} = 8.5$ (m) B1ft

$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{ their (b) answer}}$;

$\Rightarrow t = \text{awrt } 4.41$ M1; A1 3

Note

B1$\sqrt{\ }$: 8.5 or 6 + their R found in part (a) as long as the answer is greater than 6.
M1: sin $\left(\frac{4\pi t}{25} \pm \textit{their}\ \alpha\right)$ = 1 or $\frac{4\pi t}{25}$ = their (b) answer
A1: For $\sin^{-1}$ (0.4) This can be implied by awrt 4.41 or awrt 4.40.

(d) $\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$;

$\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$ M1; M1

$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41 A1

Either t = awrt 2.1 or awrt 6.7 A1

So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^{c}\right\}$ ddM1

Times = $\{14:06,\ 18:43\}$ A1 6

Note

M1: 6 + (their R) sin $\left(\frac{4\pi t}{25} \pm \textit{their}\ \alpha\right)$ = 7, M1:
$\left(\frac{4\pi t}{25} \pm \textit{their}\ \alpha\right) = \frac{1}{\textit{their}\ R}$
A1: For $\sin^{-1}$ (0.4). This can be implied by awrt 0.41 or awrt 2.73 or other values for different α ‘s. Note this mark can be implied by seeing 1.055.
A1: Either t = awrt 2.1 or t = awrt 6.7
ddM1: either π – their PV^{c}. Note that this mark is dependent upon the two M marks.
This mark will usually be awarded for seeing either 2.730… or 3.373…
A1: Both t = 14 : 06 and t = 18 : 43 or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.

[15]

3.	(a)	$\cos^2\theta + \sin^2\theta = 1\ (\div \cos^2\theta)$			
		$\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1	
		$1 + \tan^2\theta = \sec^2\theta$			
		$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen.	A1 cso	2
	(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$, (eqn *) $0 \le \theta < 360°$			
		$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1	
		$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$			
		$\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1	
		$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1	
		$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$			
		$\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$			
		$\underline{\cos\theta = -\tfrac{1}{2}}$; or $\cos\theta = \tfrac{3}{2}$	$\underline{\cos\theta = -\tfrac{1}{2}}$;	A1;	
		$\alpha = 120°$ or α = no solutions			
		$\theta_1 = \underline{120°}$	$\underline{120°}$	$\underline{\text{A1}}$	
		$\theta_2 = 240°$	$\underline{240°}$ or $\theta_2 = 360° - \theta_1$ when solving using $\cos\theta = ...$	B1ft	6
		$\theta = \{120°, 240°\}$	Note the final A1 mark has been changed to a B1 mark.		

[8]

4.	(a)	$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A + B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1	
		$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives			
		$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required) Complete proof, with a link between LHS and RHS. No errors seen.		A1 **AG**	2

(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1	
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1	
	$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$			
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$			
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 **AG**	3
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$			
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$			
	Equate $\sin 2x$: $3 = R\sin\alpha$			
	Equate $\cos 2x$: $4 = R\cos\alpha$			
	$R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$	$R = 5$	B1	
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...°$	$\sin\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1	
		awrt 36.87	A1	3
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$			
(d)	$3\sin 2x + 4\cos 2x = 2$			
	$5\cos(2x - 36.87) = 2$			
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1	
	$(2x - 36.87) = 66.42182...°$	awrt 66	A1	
	$(2x - 36.87) = 360 - 66.42182...°$			
	Hence, $x = 51.64591...$o, $165.22409...°$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1	
		Both awrt 51.6 AND awrt 165.2	A1	4
		If there are any EXTRA solutions inside the range $0 \leq x < 180°$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \leq x < 180°$.		

[12]

5. (a) $\sin 2x = \underline{2\sin x\cos x}$ B1 aef 1

(b) $\operatorname{cosec} x - 8\cos x = 0,\ 0 < x < \pi$

$\frac{1}{\sin x} - 8\cos x = 0$ Using $\operatorname{cosec} x = \frac{1}{\sin x}$ M1

$\frac{1}{\sin x} = 8\cos x$

$1 = 8\sin x\cos x$

$1 = 4(2\sin x\cos x)$

$1 = 4\sin 2x$

$\underline{\sin 2x = \tfrac{1}{4}}$ $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$ M1

$\underline{\sin 2x = \tfrac{1}{4}}$ A1

Radians $2x = \{0.25268..., 2.88891...\}$

Degrees $2x = \{14.4775..., 165.5225...\}$

Radians $x = \{0.12634..., 1.44445...\}$

Degrees $x = \{7.23875..., 82.76124...\}$

Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. A1

Both $\underline{0.13}$ and $\underline{1.44}$ A1 cao 5

Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.

[6]

6. (a) (i) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ M1 A1

$= 2\sin\theta\,(1 - \sin2\theta) + \sin\theta - 2\sin^3\theta$ M1

$= 3\sin\theta - 4\sin^3\theta$ * cso A1 4

(ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$

$-2\sin 3\theta + 1 = 0$ M1 A1

$\sin 3\theta = \frac{1}{2}$ M1

$3\theta \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{18}, \frac{5\pi}{6}$ A1 A1 5

(b) $\sin 15° = \sin(60° - 45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$ M1

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ M1 A1

$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso A1 4

Alternatives

1 $\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$ M1

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ M1 A1

$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso A1 4

2 Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$

$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$

$\sin^2 15° = \frac{2 - \sqrt{3}}{4}$ M1 A1

$\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$ M1

Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso A1 4

[13]

7.	(a)	$\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$		B1	
		$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$		B1B1	
		$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$		M1	
		$= 3\sin\theta - 4\sin^3\theta$ *	cso	A1	5
	(b)	$\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$	or exact	M1A1	2
		equivalent			

[7]

8.	(a)	Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule]	M1	
		$DE = 4\sin\theta$ (*) (c.s.o.)	A1(*)	2
	(b)	$P = 2\,DE + 2EF$ or equivalent. With attempt at EF	M1	
		$= 8\sin\theta + 4\cos\theta$ (*) (c.s.o.)	A1(*)	2
	(c)	$8\sin\theta + 4\cos\theta = R\sin(\theta + \alpha)$		
		$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$		
		Method for R, method for α	M1 M1	
		need to use tan for 2nd M		
		[$R\cos\alpha = 8$, $R\sin\alpha = 4$ $\tan\alpha = 0.5$, $R = \sqrt{(8^2 + 4^2}$]		
		$R = 4\sqrt{5}$ or 8.94, $\alpha = 0.464$ (allow 26.6),	A1 A1	4
		awrt 0.464		
	(d)	Using candidate's $R\sin(\theta + \alpha) = 8.5$ to give $(\theta + \alpha) = \sin^{-1}\frac{8.5}{R}$	M1	
		Solving to give $\theta = \sin^{-1}\frac{8.5}{R} - \alpha$, $\theta = 0.791$ (allow 45.3)	M1 A1	
		Considering second angle: $\theta + \alpha = \pi$ (or 180) $- \sin^{-1}\frac{8.5}{R}$;	M1	
		$\theta = 1.42$ (allow 81.6)	A1	5

[13]

9. (i) A correct form of cos $2x$ used M1

$1-2\left(\frac{3}{5}\right)^2$ or $\left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$ or $2\left(\frac{4}{5}\right)^2-1$ $\left\{\frac{7}{25}\right\}$ A1

$\sec 2x = \frac{1}{\cos 2x}$; $= \frac{25}{7}$ or $3\frac{4}{7}$ M1A1 4

(ii) (a) $\frac{\cos 2x}{\sin 2x}+\frac{1}{\sin 2x}$ or (b) $\frac{1}{\tan 2x}+\frac{1}{\sin 2x}$ M1

Forming single fraction (or ** multiplying both sides by sin2x) M1

Use of correct trig. formulae throughout and producing expression in terms of sin$x$ and cos$x$ M1

Completion (cso) e.g. $\frac{2\cos^2 x}{2\sin x\cos x} = \frac{\cos x}{\sin x} = \cot x$ (*) A1 4

[8]

10. (a) $e^w = 0.25 \Rightarrow w = -1.39$ M1 A1 2

(b) $\arctan x = 1 \Rightarrow x = 0.79$ M1 A1 2

(c) $\ln \frac{y+1}{y} = 0.85 \Rightarrow \frac{y+1}{y} = e^{0.85}$ M1 A1

$\frac{1}{y} = 2.340 - 1 \Rightarrow y = 0.75$ M1 A1 4

(d) Putting cos z + sin z in the form $\sqrt{2}\cos\left(z-\frac{\pi}{4}\right)$ or equivalent M1 A1

$\cos\left(z-\frac{\pi}{4}\right) = \frac{1}{3\sqrt{2}}$

attempt for z M1

$z = 2.12, -0.55$ A1, A1 ft 5

[13]

1. This proved to be a fairly friendly opening question with about 45% of candidates gaining all 5 marks, with only about 10% of candidates unable to score.

In part (a), a majority of candidates were able to use both a correct identity for sin 2θ as 2 sin θ cos θ **and** a correct identity for cos θ as 2 $\cos^2 \theta - 1$ and were usually successful in their proof. There were a small minority of candidates who correctly replaced 1 + cos 2θ as 2 $\cos^2 \theta$ and thus achieved the given result with ease. Those candidates who were less successful used correct identities for cos 2θ as either $\cos^2 \theta - \sin^2 \theta$ or $1 - 2\sin^2 \theta$but failed to realise that they needed to apply the identity $\cos^2 \theta + \sin^2 \theta = 1$ in order to proceed to the correct result.

In part (b), a majority of candidates were able to make a link with part (a), to arrive at the equation tan $\theta = \frac{1}{2}$ with many giving both correct angles of 26.6° and –153.4° in the range $-180° \leq \theta < 180°$. There were a significant minority of candidates who either only wrote down 26.6° or gave 206.6° as their second angle or gave extra solutions such as –26.6° or 153.4°. A significant number of candidates, surprisingly wrote down tan $\theta = 1$ and proceeded in most cases to give 45° and –135°.

2. This was the most demanding question on the paper and many candidates were unable to apply their successful work in parts (a) and (b) to the other two parts of the question. The mean mark for this question was 8.3 and about 12% of the candidates scored all 15 marks.

In part (a), almost all candidates were able to obtain the correct value of R, although a few omitted it at this stage and found it later on in the question. Some candidates incorrectly wrote tanα as either $\frac{2}{1.5}, -\frac{2}{1.5}$ or $-\frac{1.5}{2}$. In all of these cases, such candidates lost the final accuracy mark for this part. A significant number of candidates found α in degrees, although many of them converted their answer into the required radian answer.

In part (b), many candidates were able to state the maximum value. A significant number of candidates wrote down incorrect equations such as $2.5\sin(\theta - \alpha) = 1$ or $2.5(\sin(\theta - \alpha) = 0$ in order to find θ. Few candidates attempted a calculus method and although some proceeded to achieve tan$\theta = \pm k$ it was rare for them to find the correct answer of 2.21^c.

Many candidates failed to make a connection between part (c) and part (b). These candidates only worked with the given expression and assumed that the maximum occurred when $\sin\left(\frac{4\pi t}{25}\right) = 1$ and $\cos\left(\frac{4\pi t}{25}\right) = 0$. Of those who made the connection, many wrote down the correct maximum of 8.5 and solved $H = 8.5$ to achieve $\sin\left(\frac{4\pi t}{25} + \text{their}\,\alpha\right) = 1$ or made the link with part (b) to write $\frac{4\pi t}{25} = 2.214$ A significant number of candidates incorrectly solved $\frac{4\pi t}{25} = 2.214$ to give $t = 43.47$. Failure to correctly use a calculator correctly was apparent with dividing by 4π being processed by their calculator as "divide by 4 and multiply by π". Again, a calculus method was rarely seen and applied with little success.

Part (d) was a good source of some marks and was frequently well attempted by those candidates who had failed to make any headway with part (c). Again the "calculator error" lost candidates marks, with $t = 20.71$ being seen on a number of occasions.

Disappointingly, only a minority of candidates recognised the need for a second solution and so lost the final two marks. Some candidates did not appreciate that t was measured in hours and gave their answers as 2 minutes and 7 minutes past midday. Most candidates worked in radians, but those working in degrees usually tried to solve the equation

$6 + 2.5\sin\left(\frac{4\pi t}{25} + 36.8699°\right) = 7$ and so lost many marks.

3. In part (a), the majority of candidates started with $\cos^2\theta + \sin^2\theta = 1$ and divided all terms by $\cos^2\theta$ and rearranged the resulting equation to give the correct result. A significant minority of candidates started with the RHS of $\sec^2\theta - 1$ to prove the LHS of $\tan^2\theta$ by using both $\sec^2\theta = \frac{1}{\cos^2\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$. There were a few candidates, however, who used more elaborate and less efficient methods to give the correct proof.

In part (b), most candidates used the result in part (a) to form and solve a quadratic equation in $\sec\theta$ and then proceeded to find 120° or both correct angles. Some candidates in addition to correctly solving $\sec\theta = -2$ found extra solutions by attempting to solve $\sec\theta = \frac{2}{3}$, usually by proceeding to write $\cos\theta = \frac{2}{3}$, leading to one or two additional incorrect solutions. A significant minority of candidates, however, struggled or did not attempt to solve $\sec\theta = -2$.

A significant minority of candidates used $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$ to achieve both answers by a longer method but some of these candidates made errors in multiplying both sides of their equation by $\cos^2\theta$.

4. The majority of candidates were able to give a correct proof in part (a). A number of candidates having written $\cos 2A = \cos^2 A - \sin^2 A$ did not make the connection with $\sin^2 A + \cos^2 A = 1$ and were unable to arrive at the given result.

Part (b) proved to be one of the most challenging parts of the paper with many candidates just gaining the first mark for this part by eliminating y correctly. A number of candidates spotted the link with part (a) and either substituted $\frac{1-\cos 2x}{2}$ for $\sin^2 x$ or $1 - \cos^2 x$ for $2\sin^2 x$ and usually completed the proof in a few lines. A significant number of candidates manipulated $4\sin^2 x - 2\cos^2 x$ to $8\sin^2 x - 2$ and usually failed to progress further. There were some candidates who arrived at the correct result usually after a few attempts or via a tortuous route.

Part (c) was well done. R was usually correctly stated by the vast majority of candidates. Some candidates gave α to 1 decimal place instead of the 2 decimal places required in the question. Other candidates incorrectly wrote $\tan\alpha$ *as* $\frac{4}{3}$. In both cases, such candidates lost the final accuracy mark for this part. There was some confusion between $2x$ and α, leading to some candidates writing $\tan 2x$ *as* $\frac{3}{4}$ and thereby losing the two marks for finding α

Many candidates who were successful in part (c) were usually able to make progress with part (d) and used a correct method to find the first angle. A number of candidates struggled to apply a correct method in order to find their second angle. A significant number of candidates lost the final accuracy mark owing to incorrect rounding errors with either one or both of 51.7° or 165.3° seen without a more accurate value given first.

5. In part (a), most candidates were able to write down the correct identity for sin $2x$.

In part (b), there was a failure by a significant number of candidates who replaced cosec x with $\frac{1}{\sin x}$ to realise the connection between part (a) and part (b) and thus managed only to proceed as far as $1 = 8\sin x \cos x$. Some candidates, however, thought that $8\sin x \cos x$ could be written $\sin 8\,x$, presumably by continuing the imagined "pattern" with $2\sin x \cos x = \sin 2x$. Nonetheless, the majority of candidates who reached this stage usually used the identity in part (a) to substitute $4\sin 2x$ for $8\sin x \cos x$ and proceeded to give at least one allowable value for x.

A number of candidates lost the final accuracy mark for only giving one instead of two values for x, or for rounding one of their answers in radians incorrectly (usually by writing 1.45 instead of 1.44). Several candidates lost the final accuracy mark for writing their answers in degrees rather than radians. Some candidates, however, worked in degrees and converted their final answers to radians.

6. Part (a)(i) was well done and majority of candidates produced efficient proofs. Some candidates, however, failed to gain full marks when the incorrect use of, or omission of, brackets led to incorrect manipulation. Those who failed to spot the connection between parts (a)(i) and (a)(ii) rarely made any progress. Those who did make the connection often made sign errors and the incorrect equation $\sin\ 3\theta = -\frac{1}{2}$ was commonly seen. The majority of those who obtained the correct $\sin\ 3\theta = \frac{1}{2}$ did obtain the two answers in the appropriate range and the instruction to give the answers in terms of π was well observed.

Many candidates struggled with part (b) and, despite the hint in the question, blank responses were quite common. Those who did attempt to write 15° as the difference of two angles often chose an inappropriate pair of angles, such as 75° and 60°, which often led to a circular argument. If an appropriate pair of angles were chosen, those who used sin 45° cos 45° = $\frac{\sqrt{2}}{2}$ usually found it easier to complete the question than those who used sin45° cos 45° = $\frac{1}{\sqrt{2}}$.

7. The great majority of candidates were able to expand $\sin(2\theta + \theta)$ correctly and replace $\sin 2\theta$ by $2\sin\theta\cos\theta$. However the identity $\cos 2\theta = –2\sin^2\theta$ seemed less well known and this often led to inaccurate or, more frequently, to unnecessarily lengthy proofs. Errors sometimes arose due to incorrect bracketing, $(1 – 2\sin^2\theta)\sin\theta$ being written as $1 – 2\sin^2\theta\sin\theta$. However, fully correct solutions to part (a) were common. Part (b) was also well done but, as noted in the introduction above, there were candidates who thought that a decimal answer from a calculator would be acceptable. The commonest error seen in exact manipulation was to evaluate $4\left(\frac{\sqrt{3}}{4}\right)^3$ as 3√3, not recognising that the cube applied to the 4 as well as the √3.

8. Parts (a) & (b) were poorly done with many candidates not getting (a) but working backwards to get (b). Much valuable time was wasted, often writing a page or more to gain 1 or 2 of the marks.

Part (c) was generally well done, with the main error being α being given in degrees rather the required radians. In Part (d) most candidates gained one value for x, but either did not work out the second one or incorrectly used π – first one. Accuracy marks were also lost in both (c) and then (d) by students not using accurate answers in follow through work.

9. Only the more able candidates produced concise correct solutions to this question. In part (i) candidates were required to use an appropriate double angle formula; finding x from the calculator and then substituting the result in sec $2x$ = $\frac{1}{\cos 2x}$ only gained one mark. The other most common error seen was to evaluate $\left\{1 - 2\sin^2\left(\frac{3}{5}\right)\right\}$ instead of $\left\{1 - 2\left(\frac{3}{5}\right)^2\right\}$ for cos2x, but $\frac{1}{1 - 2\sin^2 x}$ becoming 1 – ½ $\sin^2 x$ was also noticed too often In part (ii) candidates who rewrote cot $2x$ as $\frac{\cos 2x}{\sin 2x}$, rather than $\frac{1}{\tan 2x}$ and then $\frac{1 - \tan^2 x}{2\tan x}$, made the most progress but it was disappointing to see the problems this part caused and the amount of extra space that many candidates required.

10. No Report available for this question.